QP CODE: 26023303

Reg No : .....

Name : .....

B.Sc DEGREE (CBCS)SPECIAL RE APPEARANCE EXAMINATIONS, MAY 2026

(EXCLUSIVELY FOR THE FAILED CANDIDATES OF 2023 ADMISSION)

Sixth Semester

CORE COURSE - MM6CRT04 - LINEAR ALGEBRA

Common for B.Sc Mathematics Model I & B.Sc Mathematics Model II Computer Science

2023 Admission Only

4D0FC57C

Time: 3 Hours Max. Marks : 80

core

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. a)Define linearly independent rows.

   b)Prove that in the matrix $\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$ *the rows are linearly independent*

2. *Prove that the row rank of a matrix is the number of non-zero rows in any row-echelon form of the matrix.*

3. Define a vector space and give an example of a vector space

4. *Prove that* $L = \{(x,y): \alpha x + \beta y = 0\}$ *is a subspace of* R^2

5. Define a basis of a vector space V and Prove that $\{(1,1), (1,-1)\}$ is a basis of R^2.

6. Define a linear transformation from a vector space to a vector space.

7. State Dimension Theorem for finite-dimensional vector spaces.

8. Define an ordered basis of a vector space. Prove that every basis of n elements give rise to $n!$ distinct ordered bases.

9. Define a nilpotent linear mapping f on a vector space V of dimension n over a field F. What is meant by index of nilpotency of f.

10. Define eigen value and eigen vector of a matrix.

11. Define the eigen space and geometric multiplicity associated with the eigen value.

12. Define eigen value of a linear map and the eigen vector associated with it.

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. a)Prove that every square matrix can be expressed uniquely as the sum of a symmetric matrix and a skew symmetric matrix

b)If $A = \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix}$ Prove that $A^n = \begin{bmatrix} cosn\theta & sinn\theta \\ -sinn\theta & cosn\theta \end{bmatrix}$

14. a) If A and B are orthogonal nxn matrices prove that AB is orthogonal.

b) Prove that a real 2x2 matrix is orthogonal if and only if it is of one of the forms $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}$, $\begin{bmatrix} a & b \\ b & -a \end{bmatrix}$ Where $a^2 + b^2 = 1$.

15. Find the inverse of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 4 \\ 1 & 4 & 4 \end{bmatrix}$

16. *Determine which of following subsets of M_{3x1} R are linearly dependent*

a) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} \right\}$ b) $\left\{ \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \right\}$

17. a) Define rank and nullity of a linear mapping. Find the rank and nullity of $pr_1 : \mathbb{R}^3 \to \mathbb{R}$ defined by $pr_1(x, y, z) = x$.

b) Let V and W be vector spaces each of dimension n over a field F. If $f : V \to W$ is linear, then prove that f is injective if and only if f is bijective.

18. a) Prove that a linear mapping is completely and uniquely determined by its action on a basis.

b) Prove that two linear mappings $f, g : V \to W$ are equal if and only if they agree on any basis of V.

19. a) Define transition matrix from the basis $(v_i)_m$ to the basis $(v_i')_m$ of a vector space V.

b) Determine the transition matrix from the ordered basis $\{(1, 0, 0, 1), (0, 0, 0, 1), (1, 1, 0, 0), (0, 1, 1, 0)\}$ of $\mathbb{R}^4$ to the natural ordered basis of $\mathbb{R}^4$.

c) Determine the transition matrix from the ordered basis $\{(1, -1, 1), (1, -2, 2), (1, -2, 1)\}$ of $\mathbb{R}^3$ to the natural ordered basis of $\mathbb{R}^3$..

20. For the matrix $\begin{bmatrix} 3 & -1 & 0 \\ -1 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ determine an invertible matrix P such that $P^{-1}AP$ is diagonal.

21. For the nXn tridiagonal matrix An = $\begin{bmatrix} 2 & 1 & 0 & 0 & \dots & 0 & 0 \\ 1 & 2 & 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & 2 & 1 & \dots & 0 & 0 \\ . & . & . & . & \dots\dots & . & . \\ 0 & 0 & 0 & 0 & \dots & 2 & 1 \\ 0 & 0 & 0 & 0 & \dots & 1 & 2 \end{bmatrix}$ Prove that det An = n + 1.

(6×5=30)

Part C

*Answer any **two** questions.*

*Each question carries **15** marks.*

22. a) Reduce the following matrix to Hermite form $\begin{bmatrix} 1 & 2 & 1 & 2 & 1 \\ 2 & 4 & 4 & 8 & 4 \\ 3 & 6 & 5 & 7 & 7 \end{bmatrix}$

b) Prove that by using elementary row operation, a non-zero matrix can be transformed to a row-echelon matrix.

c) Prove that every non-zero matrix A can be transformed to a Hermite matrix by using elementary row operations

23. a)Prove that the set W of complex matrices of the form $\begin{bmatrix} \alpha & \beta \\ -\bar{\beta} & -\alpha \end{bmatrix}$ is a real vector space of dimension 4.

b)If V is a vector space over C of dimension n, prove that V is a vector space over R of dimension 2n .

c) If S is a subset of V then prove that S is a basis if and only if S is a maximal independent subset.

24. a) Define $Im\ f$ and $Ker\ f$ where f is a linear mapping from a vector space to a vector space.

b) Write image and kernel for $f_A : Mat_{n\times 1}\,\mathbb{R} \to Mat_{n\times 1}\,\mathbb{R}$ described by $f_A(\mathbf{x}) = A\mathbf{x}$ where A is a given real $n \times n$ matrix.

c) For the linear mapping $f : \mathbb{R}^4 \to \mathbb{R}^3$ given by $f(a, b, c, d) = (a + b,\ b - c,\ a + d)$, find $Im\ f$. Also find a basis for $Im\ f$ and prove that $Im\ f = \mathbb{R}^3$.

25. a) Define similar matrices and state whether similar matrices have the same rank. Show that if matrices A, B are similar then so are A', B'.

b) Check whether for every $\vartheta \in \mathbb{R}$, the complex matrices $\begin{bmatrix} cos\,\vartheta & sin\,\vartheta \\ -sin\,\vartheta & cos\,\vartheta \end{bmatrix}$, $\begin{bmatrix} e^{i\vartheta} & 0 \\ 0 & e^{-i\vartheta} \end{bmatrix}$ are similar.

c) Prove that the relation of being similar is an equivalence relation on the set of $n \times n$ matrices.

(2×15=30)